Problem 1 **CMWMC 2022, Relay Round, Set 1/4**

Compute the number of real numbers x such that the sequence $x, x^2, x^3, x^4, x^5, \ldots$ eventually repeats. (To be clear, we say a sequence "eventually repeats" if there is some block of consecutive digits that repeats past some point—for instance, the sequence $1, 2, 3, 4, 5, 6, 5, 6, 5, 6, \ldots$ is eventually repeating with repeating block $5, 6$.)

Problem 2 **CMWMC 2022, Relay Round, Set 1/4**

Let T be the answer to the previous problem. Nicole has a broken calculator which, when told to multiply a by b, starts by multiplying a by b, but then multiplies that product by b again, and then adds b to the result. Nicole inputs the computation "$k \times k$" into the calculator for some real number k and gets an answer of $10T$. If she instead used a working calculator, what answer should she have gotten?

Problem 3 **CMWMC 2022, Relay Round, Set 1/4**

Let T be the answer to the previous problem. Find the positive difference between the largest and smallest perfect squares that can be written as $x^2 + y^2$ for integers x, y satisfying $\sqrt{T} \leq x \leq T$ and $\sqrt{T} \leq y \leq T$.

Problem 1 **CMWMC 2022, Relay Round, Set 2/4**

What is the last digit of $2022 + 2022^{2022} + 2022^{(2022^{2022})}$?

Problem 2 **CMWMC 2022, Relay Round, Set 2/4**

Let T be the answer to the previous problem. CMIMC executive members are trying to arrange desks for CMWMC. If they arrange the desks into rows of 5 desks, they end up with 1 left over. If they instead arrange the desks into rows of 7 desks, they also end up with 1 left over. If they instead arrange the desks into rows of 11 desks, they end up with T left over. What is the smallest possible (non-negative) number of desks they could have?

Problem 3 **CMWMC 2022, Relay Round, Set 2/4**

Let T be the answer to the previous problem. Compute the largest value of k such that 11^k divides

$$T! = T(T-1)(T-2)\dots(2)(1).$$

Problem 1 **CMWMC 2022, Relay Round, Set 3/4**

Annie has 24 letter tiles in a bag; 8 C's, 8 M's, and 8 W's. She blindly draws tiles from the bag until she has enough to spell "CMWMC." What is the maximum number of tiles she may have to draw?

Problem 2 **CMWMC 2022, Relay Round, Set 3/4**

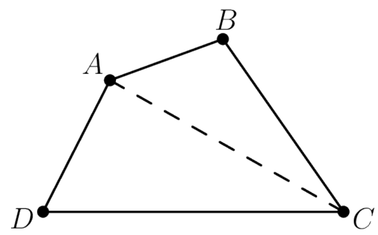
Let T be the answer from the previous problem. Charlotte is initially standing at $(0, 0)$ in the coordinate plane. She takes T steps, each of which moves her by 1 unit in either the $+x$, $-x$, $+y$, or $-y$ direction (e.g. her first step takes her to $(1, 0)$, $(1, 0)$, $(0, 1)$ or $(0, -1)$). After the T steps, how many possibilities are there for Charlotte's location?

Problem 3 **CMWMC 2022, Relay Round, Set 3/4**

Let T be the answer from the previous problem, and let S be the sum of the digits of T. Francesca has an unfair coin with an unknown probability p of landing heads on a given flip. If she flips the coin S times, the probability she gets exactly one head is equal to the probability she gets exactly two heads. Compute the probability p.

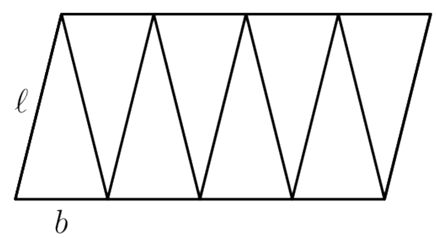
Problem 1 **CMWMC 2022, Relay Round, Set 4/4**

Quadrilateral $ABCD$ (with A, B, C not collinear and A, D, C not collinear) has $AB = 4$, $BC = 7$, $CD = 10$, and $DA = 5$. Compute the number of possible integer lengths AC.



Problem 2 **CMWMC 2022, Relay Round, Set 4/4**

Let T be the answer from the previous part. $2T$ congruent isosceles triangles with base length b and leg length ℓ are arranged to form a parallelogram as shown below (not necessarily the correct number of triangles). If the total length of all drawn line segments (**not** double counting overlapping sides) is exactly three times the perimeter of the parallelogram, find $\frac{\ell}{b}$.



Problem 3 **CMWMC 2022, Relay Round, Set 4/4**

Let T be the answer from the previous part. Rectangle R has length T times its width. R is inscribed in a square S such that the diagonals of S are parallel to the sides of R. What proportion of the area of S is contained within R?